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String breaking in quenched QCD

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We present results on a new operator for the investigation of string-breaking effects in quenched SU(2)-color QCD. The ground state of a spatially separated static-light meson-antimeson pair is a combination of a state with two distinct mesons, expected to dominate for large separations, and a state where the light quarks have annihilated, which contributes at short distances. The crossover between these two regimes provides the string-breaking scale. [S0556-2821(99)05911-1]

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I. INTRODUCTION

An early success of lattice QCD simulations was the first-principles demonstration that the potential between static quarks is confining. The binding energy in the static quark-pair system is described very well by a Coulomb-plus-linear potential. Recent studies have confirmed the expectation that the gauge field forms a narrow flux tube, or string, joining the static quarks [1].

In unquenched simulations, we expect the static-quark potential to show evidence for string-breaking—when the energy in the gauge field string allows the creation of a light quark pair from the vacuum, the system should ''break'' into a static-light meson-antimeson pair. The signal of string-breaking would be a plateau in the static-quark potential, around twice the mass of the static-light meson.

A first-principles demonstration of string-breaking has so far eluded lattice QCD practitioners. Traditionally, researchers have performed Wilson loop simulations in full QCD, searching for the signature plateau in the static quark potential. Recent results show no evidence for this effect [2,3]. Using a method for separating high and low eigenmodes of the quark operator, Duncan et al. report evidence for screening effects at short distances [4], though their lattice dimensions are too small to observe a crossover into the plateau region.

This has led some to suggest that the Wilson loop operator may have too small an overlap with the broken two-meson state [2,5], and to recommend a search for better operators. Theoretical investigations have widely confirmed this stance, and point to the utility of a mixed-operator approach to numerical simulations of string breaking [6–8]. Simulations involving hybrids of Wilson loop and meson-pair operators have been highly successful in scalar models [9–11], and investigations of four-quark systems using pseudofermion techniques look very promising [12].

In a previous paper, we described a derivation of the binding potential between two static-light mesons in the quenched approximation of SU(2) QCD [13]. A similar investigation of the binding potential in a static meson-antimeson pair was abandoned due to difficulties with the operator's short-range behavior—the derived binding potential appeared identical to the Wilson loop potential, which we interpreted as a signal that the light quarks were annihilating, leaving only a static quark pair.

The light-quark annihilation led us to conclude that the meson-antimeson operator, while perhaps unsuitable for an investiagtion of the binding potential, might be ideal for a demonstration of string-breaking phenomena. This approach is a reversal of the usual Wilson loop approach—we know that our operator has the correct long-distance behavior, and wish to show it also describes the unbroken-string short-distance behavior of the Wilson loop.

Our aim in this paper is to demonstrate that the static-light meson-antimeson operator is well-suited to string-breaking applications, providing superior overlap with the broken-string state while retaining the necessary overlap with the unbroken quark-pair state. We describe the operator and the simulation parameters in the following sections, and then present preliminary results that indicate the operator does describe string-breaking within the quenched approximation. This result makes us confident that the operator will be useful in full-QCD simulations.

II. THE OPERATOR

The standard operator used in string-breaking investigations is the Wilson loop, which for a closed loop $C(R,T)$ of spatial width R and time extent T is

$$W(R,T)=\mathrm{Tr}\prod_{l\in C_{R,T}} U_l . \tag{1}$$

The Wilson loop is the propagator for a spatially-separated static quark-antiquark pair. By design, this operator has strong overlap with the unbroken state of two static quarks joined by a gluon flux tube—sadly, it has proven to have insufficient overlap with the broken state of two distinct static-light mesons, as recently demonstrated by Knechtli and Sommer [9]. The lack of evidence for string-breaking is most likely due to this poor overlap with the broken state: the meson-pair state simply isn't ''seen'' by the Wilson loop operator.

Consider then, a composite operator consisting of a static-light meson-antimeson pair, separated by a distance $\vec{R}$,

$$\mathcal{O}(\vec{R})=\overline{\psi_l}(0)\Gamma\psi_h(0)\overline{\psi_h}(\vec{R})\Gamma^\dagger\psi_l(\vec{R}). \tag{2}$$

We used a smeared-source meson operator, with

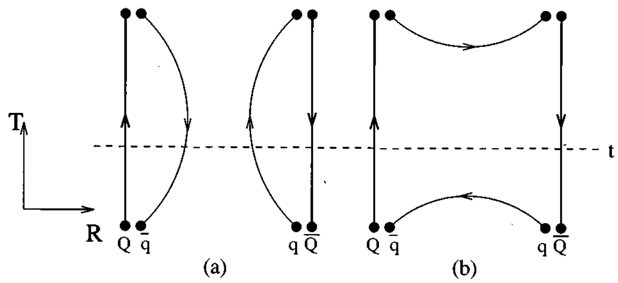

FIG. 1. Contributions to (a) direct and (b) exchange terms. Note the light-quark annihilation in (b)—at the time slice indicated by the dashed line, only the static quarks remain.

$$\Gamma = \gamma_5 (1 + \epsilon \Delta^2)^{n_s} \tag{3}$$

to improve the overlap with the meson ground state [14]. The meson-pair correlator is

$$G(t,\vec{R}) = \mathcal{G}_D + \mathcal{G}_E, \tag{4}$$

where

$$\begin{aligned}\mathcal{G}_D(t,\vec{R}) &= \mathrm{Tr}[G_h(0,t;0,0) G_l^\dagger(0,t;0,0)] \\ &\quad \times \mathrm{Tr}[G_l(\vec{R},t;\vec{R},0) G_h^\dagger(\vec{R},t;\vec{R},0)], \\ \mathcal{G}_E(t,\vec{R}) &= -\mathrm{Tr}[G_h(0,t;0,0) G_l^\dagger(0,0;\vec{R},0) \\ &\quad \times G_h^\dagger(\vec{R},t;\vec{R},0) G_l(0,t;\vec{R},t)].\end{aligned} \tag{5}$$

Contributions to $\mathcal{G}_D$ and $\mathcal{G}_E$, the ''direct'' and ''exchange'' terms, are depicted in Fig. 1.

In the large-R limit, the operator describes two distinct static-light mesons, and so we expect $\mathcal{G}_D$ will dominate the correlator for large separations. For small R, however, the light quarks can easily annihilate, as shown in Fig. 1(b), leaving a static quark-antiquark pair interacting through the gluon field. The exchange term $\mathcal{G}_E$ should contribute strongly for small separations. The operator in Eq. (2) provides a description of the long-distance physics of the broken mesonic state, *and* the short-distace physics of the static-quark pair thanks to the annihilation of the light quarks.

III. THE SIMULATION

We performed a lattice simulation to test the validity of Eq. (2) as a string-breaking operator. An ensemble of 344 quenched SU(2)-color gauge configurations was created using an $\mathcal{O}(a^2)$-improved action,

$$S_G = -\beta \sum_{x,\mu>\nu} \left(\frac{5}{3} \frac{P_{\mu\nu}}{u_0^4} - \frac{1}{12} \frac{R_{\mu\nu} + R_{\nu\mu}}{u_0^6} \right), \tag{6}$$

where $P_{\mu\nu}$ is the plaquette operator, and $R_{\mu\nu}$ is a 2×1 loop with the long side along the μ-direction. The lattice dimensions were $(L_x, L_y, L_z, L_t) = (10,8,8,12)$, with the separation between the static quarks along the x-direction. The tadpole correction u_0 was derived from the plaquette operator,

$$u_0 = \langle P_{\mu\nu} \rangle^{1/4}. \tag{7}$$

The simulation was performed at $\beta = 1.07$, corresponding to a lattice spacing of roughly 0.2 fm, using the ρ- and π-meson mass ratio to set the scale.

We used the tadpole-improved Sheikholeslami-Wohlert operator for the fermion action,

$$M_{SW} = m_0 + \sum_\mu \left(\gamma_\mu \triangle_\mu - \frac{1}{2} \triangle_\mu^2 \right) - \frac{1}{4} \sigma \cdot F. \tag{8}$$

The correlator, Eq. (4), requires us to determine light-quark propagators for each value of R *and t*—this computational complexity forced us to choose a relatively high mass for the light quarks. We used $\kappa = 0.135$, corresponding to a pion to rho-meson mass ratio of $m_\pi / m_\rho \simeq 0.76$. We chose $\epsilon = 1/12$ and $n_s = 5$ in the smearing function, Eq. (3) [14].

TABLE I. Energy $E(R)$ in lattice units [see Eq. (9)] as a function of separation, from fits to the direct ($\mathcal{G}_D$) and exchange ($\mathcal{G}_E$) terms, the full correlator (G), and the Wilson loop potential (W). Blank entries indicate values where the signal was dominated by noise.

R	$\mathcal{G}_D(R)$	$\mathcal{G}_E(R)$	$G(R)$	$W(R)$
0	0.854(5)	0.007(8)	0.01(1)	0.0
1	0.865(6)	0.30(7)	0.53(3)	0.228(2)
2	0.876(5)	0.50(10)	0.85(1)	0.547(4)
3	0.869(5)	0.80(20)	0.89(2)	0.82(5)
4	0.870(5)		0.869(7)	1.05(20)
5	0.871(5)		0.872(7)	

IV. RESULTS

All masses and energies were taken from single cosh-squared fits to the relevant propagators, of the form

$$\mathcal{G}(R,t) \sim A \cosh^2(E(R)(t - T/2)), \tag{9}$$

where T is the temporal extent of the lattice. We found that the results from multi-cosh fits were not reliable, though this is hardly surprising given the relatively small number of propagator elements, due to the small dimensions of the lattice. The results from these fits are given in Table I, and an example fit to the raw data for the full propagator $G(t)$ and the direct term $\mathcal{G}_D(t)$ at $R=1$ is shown in Fig. 2.

Figure 3 shows a comparison of the energy, in lattice units, derived from fits to the direct and exchange terms in the propagator (4) for varying separation R. Also shown is a linear-plus-Coulomb fit to the Wilson loop potential from the same lattice ensemble. Note that for small R, the exchange term gives a contribution almost identical in size to the Wilson loop potential, indicating that the light quarks are indeed annihilating to leave a static quark-antiquark pair.

We expect string-breaking to occur at the point where the two-meson state becomes energetically favorable—from the

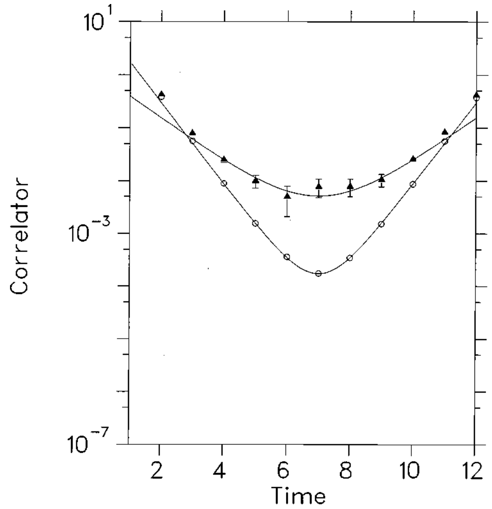

FIG. 2. The full correlator $G(R,t)$ (triangles) and the direct term $\mathcal{G}_D(R,t)$ (circles) for the meson separation $R=1$. The lines indicate fits to the data from $t=4$ to $t=10$.

slope of the Wilson loop potential, this appears to be between $R=3$ and $R=4$. Unfortunately, noise overcomes the exchange-term signal just at this point, and the crossover can only be inferred from the Wilson loop data.

The energy from fits to the full propagator, shown in Fig. 4, provides a much clearer view of the potential. The system's energy increases to a plateau at the expected level of the mass of the two distinct mesons, indicated by the horizontal lines. The static-light meson mass was taken from fits to the single-meson propagator. The small error bars on the potential for large values of R indicate that, although noise has destroyed the exchange term's signal, the mixing into this term has vanished. This is again expected, since the meson-pair state should dominate as R increases.

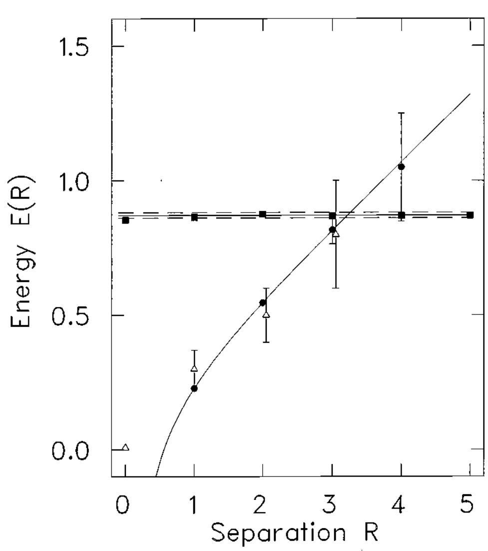

FIG. 3. Energy E(R) in lattice units [see Eq. (9)] from fits to the direct term (squares), exchange term (triangles) and Wilson loop potential (circles and solid line). Exchange term data are offset for clarity at $R=2$ and $R=3$. Horizontal lines indicate the mass of the static-light meson pair.

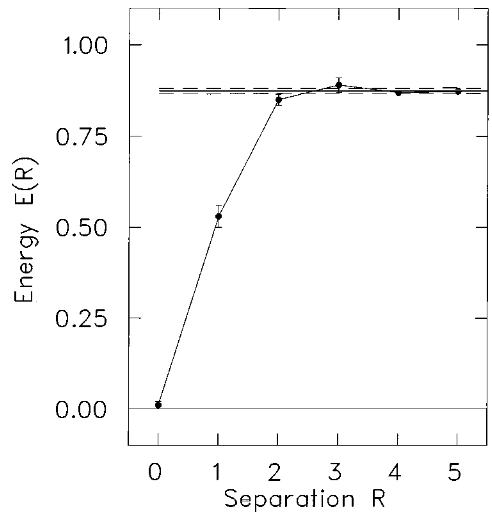

FIG. 4. Energy E(R) in lattice units [see Eq. (9)] from fits to full propagator. Horizontal lines indicate mass and error limits of the free static-light meson pair.

From Figs. 3 and 4, we estimate the string-breaking distance to be roughly $3a$, or 0.6 fermi, using our naive SU(2) lattice spacing estimate. Figure 5 supports this interpretation. The comparative sizes of $\mathcal{G}_D(t=0)$ and $\mathcal{G}_E(t=0)$ are taken as a rough measure of the mixing of each of the direct and exchange terms with the ground state, and are plotted as a function of R. As expected, the direct and exchange terms both contribute for small R, and the exchange term vanishes quickly as separation increases, leaving the direct term to dominate completely for large R.

V. CONCLUSIONS

The use of new operators, or combinations of operators, to demonstrate string-breaking on the lattice appears to be an idea whose time has arrived—witness the flood of recent papers presenting results in $SU(2)$ QCD with scalar fields

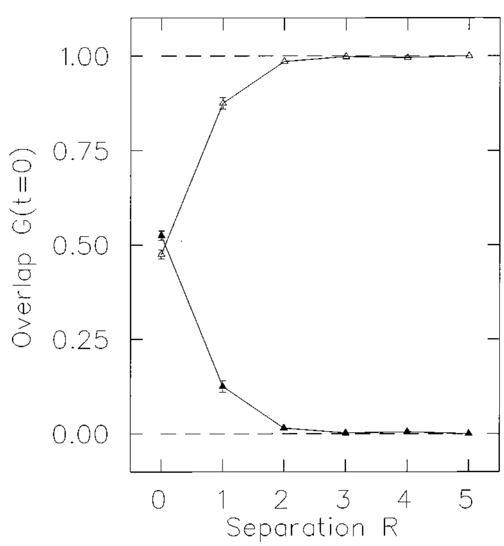

FIG. 5. $\mathcal{G}_D(t=0)$ and $\mathcal{G}_E(t=0)$ as a measure of the overlap with the ground state.

[9–11], and references to ongoing research in full $SU(3)$ QCD [1,12].

We have described an operator suitable for use in string-breaking investigations. The operator, Eq. (2), is able to describe both regimes necessary for a demonstration of string breaking—the short-range static-quark pair, and the long-range meson pair. For small separations, this operator behaves like a Wilson loop, thanks to the annihilation of the light quarks, resulting in a confining potential. For larger separations, the potential reaches a plateau at the energy of two distinct static-light mesons, the fingerprint of the elusive broken string. Our simulation gives a string-breaking distance of ~ 0.6 fm, and noting that we are working with SU(2)-color gauge fields, this result is in qualitative agreement with other estimates [9].

The string breaking occurs even in our *quenched* simulation, since the operator *must* energetically favor the meson-pair state for large *R*, and so forces the gluon string to break. The light quarks are providing, within the quenched approximation, some sea-quark effects. We expect the operator will provide the same results in unquenched simulations. Further, as stated in [7], an understanding of the role of dynamical quarks in full QCD necessarily requires a comparison between relevent quantities from quenched and unquenched simulations. In this sense, a quenched study of string breaking is useful both as a computationally simple test of the operator's utility, and as a basis for comparison with full QCD results.

Our computing resources forced certain constraints on our simulations—performing this research on a desktop workstation, we chose a high light-quark mass, small lattice volume and large lattice spacing. To counter the effects of low statistics and finite lattice spacing, we employed improved actions and operators. A truly definitive demonstration of string breaking would require much higher statistics and finer resolution than the results presented here.

We reiterate our main goal—to demonstrate the utility of the static meson-antimeson operator in string-breaking simulations. We are confident that this operator will allow accurate determination of the string-breaking distance when used in more ambitious simulations.

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